Cambridge Assessment International Education

# Cambridge International AS & A Level

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**MATHEMATICS** **9709/23**

Paper 2 Pure Mathematics 2 **October/November 2021**

**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.




**[Turn over**

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**1** Find the exact value of $\int_{-1}^{2} \left(4e^{2x} - 2e^{-x}\right) dx$. [4]

**2** **(a)** Sketch, on the same diagram, the graphs of $y = 3x$ and $y = |x - 3|$. [2]

**(b)** Find the coordinates of the point where the two graphs intersect. [3]

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**(c)** Deduce the solution of the inequality $3x < |x - 3|$. [1]

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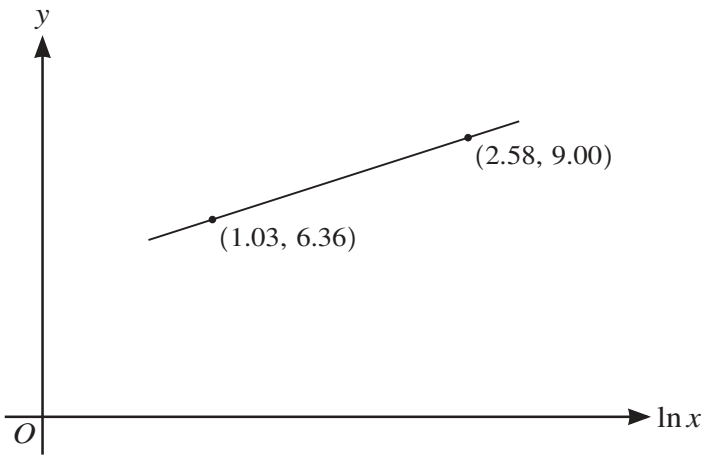
**3**




The variables $x$ and $y$ satisfy the equation $a^y = kx$, where $a$ and $k$ are constants. The graph of $y$ against $\ln x$ is a straight line passing through the points (1.03, 6.36) and (2.58, 9.00), as shown in the diagram.

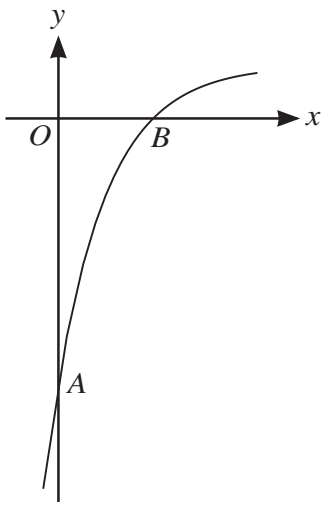
Find the values of $a$ and $k$, giving each value correct to 2 significant figures. [5]

**4** The curve with equation $y = xe^{2x} + 5e^{-x}$ has a minimum point $M$.

**(a)** Show that the $x$-coordinate of $M$ satisfies the equation $x = \frac{1}{3}\ln 5 - \frac{1}{3}\ln(1 + 2x)$. [5]

**(b)** Use an iterative formula, based on the equation in part **(a)**, to find the $x$-coordinate of $M$ correct to 3 significant figures. Use an initial value of 0.35 and give the result of each iteration to 5 significant figures. [3]

**5**




The diagram shows the curve with parametric equations

$$x = \ln(2t + 3), \qquad y = \frac{2t - 3}{2t + 3}.$$

The curve crosses the $y$-axis at the point $A$ and the $x$-axis at the point $B$.

**(a)** Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6}{2t + 3}$. [4]

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**(b)** Find the gradient of the curve at $A$. [2]

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**(c)** Find the gradient of the curve at $B$. [2]

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**6** The polynomials $f(x)$ and $g(x)$ are defined by

$$f(x) = 4x^3 + ax^2 + 8x + 15 \qquad \text{and} \qquad g(x) = x^2 + bx + 18,$$

where $a$ and $b$ are constants.

**(a)** Given that $(x + 3)$ is a factor of $f(x)$, find the value of $a$. [2]

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**(b)** Given that the remainder is 40 when $g(x)$ is divided by $(x - 2)$, find the value of $b$. [2]

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**(c)** When $a$ and $b$ have these values, factorise $f(x) - g(x)$ completely. [3]

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**(d)** Hence solve the equation $f(\text{cosec } \theta) - g(\text{cosec } \theta) = 0$ for $0 < \theta < 2\pi$. [3]

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**7** **(a)** By first expanding $\cos(2\theta + \theta)$, show that $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

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**(b)** Find the exact value of $2\cos^3\left(\frac{5}{18}\pi\right) - \frac{3}{2}\cos\left(\frac{5}{18}\pi\right)$. [2]

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**(c)** Find $\int (12\cos^3 x - 4\cos^3 3x)\,\mathrm{d}x$. [4]

## Additional Page

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